The travelling-wave plasma converter

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The travelling-wave plasma converter is analyzed allowing for the compressible nature of the fluid. The study is restricted to conditions for which the magnetic Reynolds number is small. Steady and time-varying flow conditions are investigated using small perturbation theory. Relations for optimum conversion effectiveness are established and the effects of variable electrical conductivity are investigated. Conditions for electromechanical resonance are derived and it is shown that the conversion process introduces a damping effect.

1. Introduction

Travelling magnetic waves can be used to convert kinetic energy in a moving ionized gas or plasma into electrical energy. The principle involved leads to an electrodeless conversion process which has received attention recently as one of the possible alternative approaches for large-scale generation of electricity.

So far, interest has been concentrated mainly on plasma converters employing direct current and static fields (see, for example, Rosa 1961) and experiments have already reached a very promising state for applications in the fields of electrical power generation and jet propulsion. A number of the practical difficulties associated with the d.c. converters could be avoided by using alternating electrical and gas dynamic quantities.

The general problem of electromechanical energy conversion in plasmas using alternating magnetic fields has been discussed by Barnes (1961), George & Messerle (1962), and Clark, Swift-Hook & Wright (1963), and the travelling-wave generator is described in more detail by Blake (1957), Fanucci (1962), and Jackson & Pierson (1962). In the travelling-wave converter a travelling magnetic flux wave moves in the direction of flow of the plasma as shown in figure 1. If the flux wave and plasma velocities differ, an electromechanical interaction takes place and for a fast magnetic wave the plasma is accelerated and electric energy is converted to kinetic plasma energy. When the wave velocity trails behind the plasma velocity, the conversion process reverses and kinetic energy can be converted into electrical output. The travelling-wave converter is then essentially a linear induction generator.

Electromechanical interaction is most effective when the magnetic field is at right angles to both the plasma flow velocity and the electric current in the plasma. In such a case we are dealing with the so-called crossed-field interaction. In some of the high velocity propulsion applications, or for very high electrical conductivities, this condition is not met in practice. As shown, for example, in figure 2 for a cylindrical coil arrangement the flux lines along the centre axis are parallel to the flow and some of the plasma will unavoidably slip through a narrow neck along the axis without taking part in the interaction.

A transverse travelling wave as shown in figure 1 can be produced by a sequence of transverse field windings. Three windings per wavelength are shown in the figure, but any number greater than one may be used in practice. The windings

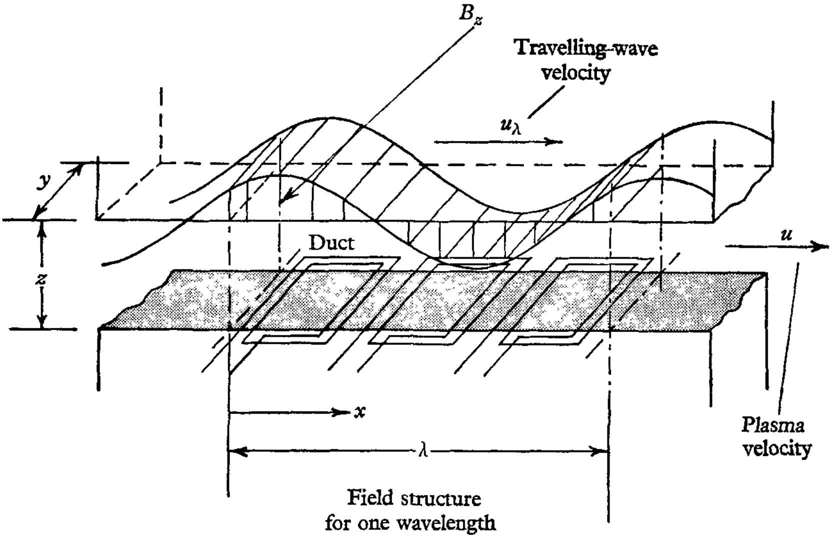

FIGURE 1. Schematic layout of travelling-wave converter showing flux wave and a stationary 3-phase field arrangement.

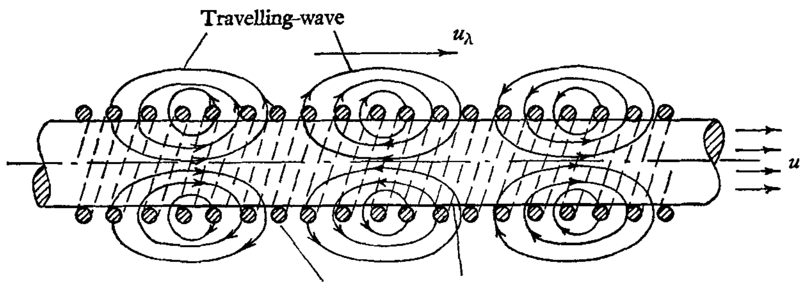

FIGURE 2. A travelling-wave converter using a co-axial field winding.

have to be equally spaced over one wavelength and each of them has to provide a flux which varies sinusoidally in time at the wave frequency f. If the flux density distribution for each winding is also sinusoidal in space, the result is a travelling transverse wave of constant amplitude as long as the time phases are properly correlated.

The travelling wave induces currents in the plasma depending on the relative velocity of wave and plasma leading to a reaction flux wave in the plasma which travels at the relative velocity. This plasma reaction flux wave corresponds to the armature reaction flux on a conventional induction machine which interacts with the stationary windings, and provides the mechanism for energy exchange between the moving plasma and the stationary windings.

Application of the travelling-wave converter principles have first been restricted to the so-called linear motor and later to the pumping of liquid metals. A fairly detailed analysis of the incompressible flow in a liquid metal pump was given by Blake (1957). Blake's study was extended later with particular reference to the generation of electrical power with gaseous fluids at high temperatures as discussed by Fanucci (1962) and Jackson & Pierson (1962). This work was mainly concerned with conditions when a relatively large electromechanical interaction is possible and in particular for relatively large magnetic Reynolds numbers. Because of the complexity of the problem the effects of compressibility of the fluid were neglected and the electrical conductivity was assumed as constant. However, the compressible nature of the flow will modify the flow conditions, and the energy conversion process must be affected by changes in electrical conductivity, which is a very sensitive function of gas temperature.

As shown by Clark *et al.* (1963), the interaction to be expected even with seeded gases will be small, in which case the distortion of the magnetic field due to any self-induced flux is small. It should be possible then to analyse the effects of compressibility and variable electrical conductivity of the fluids employing small perturbation techniques as shown for the d.c. converter by Messerle & Morrison (1962).

In this paper compressible flow conditions are studied allowing for variable electrical conductivity and the conditions of electromechanical resonance are established. Steady and time-dependent flow conditions are dealt with in turn and it is shown how optimum operation is severely modified by the compressible nature of the fluid.

2. Electrical relations for one-dimensional flow

The electrical relations at any point along the duct of a converter depend only on the local flux density $\mathbf{B}$, the electrical conductivity σ and the relative velocity $\mathbf{u}_r$ of the plasma jet with respect to the travelling wave. The relative velocity is defined as

$$\mathbf{u}_r = \mathbf{u} - \mathbf{u}_\lambda,$$

where $\mathbf{u}_\lambda$ = constant travelling-wave velocity,

and $\mathbf{u}$ = plasma velocity.

If the magnetic field is considered as a uniform wave, it is necessary to start with Maxwell's equations when contemplating a complete analysis.

The critical relations here are

$$\nabla \times \mathbf{H} = \mathbf{I} \tag{1}$$

and

$$\mathbf{I} = \sigma(\mathbf{E} + \mathbf{u}_r \times \mathbf{B}), \tag{2}$$

where $\mathbf{H}$ is the magnetic intensity, $\mathbf{I}$ the current density, $\mathbf{E}$ the voltage gradient, $\mathbf{B}$ the flux density and σ the electrical conductivity. For a relatively low electrical conductivity the current density $\mathbf{I}$ will be small. The curl relation then simplifies considerably, and the variations in current and plasma velocity across the duct become negligible. As shown by Blake (1957), this corresponds to the condition of small magnetic Reynolds number $R_M = u_r \lambda \mu \sigma$, where λ is the wavelength and μ is the magnetic permeability of the medium.

Referred to the co-ordinates indicated in figure 1, and assuming that no voltage is applied ($\mathbf{E} = 0$) and that the induced voltage is completely absorbed as resistance drop in the plasma (on a conventional induction machine this is achieved by providing short-circuiting paths alongside the duct), equation (2) becomes simply

$$\sigma^{-1} I_y = u_r B_z.$$

We neglect here viscosity effects which should be relatively small in a gaseous medium especially considering the large volumes involved in practice. The force on a volume element of thickness dx is then given by

$$dF = I_y B_z yz\, dx,$$

and the corresponding flow energy is

$$dW_c = u\, dF = u I_y B_z yz\, dx,$$

or, substituting for I_y,

$$dW_c = u u_r B_z^2 \sigma yz\, dx = u(u - u_\lambda) B_z^2 \sigma yz\, dx. \tag{3}$$

This converted flow energy is only partially available as useful output. Some of it is converted back into random heat energy by Joule heating. The rate of generation of Joule heat is

$$dW_\sigma = I_y^2 yz\, dx/\sigma, \tag{4}$$

and the electrical output follows as

$$dW_{el} = dW_c - dW_\sigma = B_z^2 \sigma yz u_\lambda (u - u_\lambda)\, dx. \tag{5}$$

The output relation (5) is positive when $u > u_\lambda$, the condition that electrical energy is generated.

3. Compressible flow relations

For incompressible flow the velocity along the duct remains constant and this effectively decouples the internal energy of the fluid from the conversion process. When allowing for compressibility, the conversion relations are then considerably modified.

For a uniform or slightly changing duct the analysis of compressible magnetohydrodynamic flow in the converter duct can be carried out in terms of one-dimensional flow equations which are the momentum equation

$$\rho \frac{du}{dt} + \frac{\partial p}{\partial x} = I_y B_z, \tag{6}$$

the energy equation

$$\rho \frac{d}{dt}(h + \tfrac{1}{2}u^2) + \frac{1}{yz}\frac{\partial W_{el}}{\partial x} = \frac{\partial p}{\partial t}, \tag{7}$$

and the continuity equation

$$\frac{\partial}{\partial t}(\rho yz) + \frac{\partial}{\partial x}(\rho u yz) = 0, \tag{8}$$

where ρ, p and h are the density, pressure and enthalpy, respectively. Additional relations required are the equation of state

$$p = \rho \mathscr{R} T, \tag{9}$$

the enthalpy equation of an ideal gas

$$h = c_p T, \tag{10}$$

and the electrical conductivity relation,

$$\sigma = \sigma(T, p), \tag{11}$$

where $\mathscr{R}$ is the gas constant and c_p the specific heat at constant pressure. Relations (6) to (8) cannot be solved in general and for the travelling-wave converter particular conditions will now be examined.

We assume that to a first approximation time variations in flow conditions produced by the flux wave are small so that time derivatives may be neglected. The momentum and energy equations then become

$$\rho u \frac{\partial u}{\partial x} + \frac{\partial p}{\partial x} + B_z^2 \sigma (u - u_\lambda) = 0, \tag{12}$$

and

$$\rho u y z \frac{\partial}{\partial x} (h + \tfrac{1}{2} u^2) + B_z^2 \, \sigma y z u_\lambda (u - u_\lambda) = 0. \tag{13}$$

These two equations can be expressed non-dimensionally as

$$\frac{\partial U}{\partial \xi} + YZ \frac{\partial P}{\partial \xi} + B^2 \Sigma \, YZ(U - E) = 0, \tag{14}$$

and

$$\frac{\partial}{\partial \xi} \left(\frac{\gamma}{\gamma - 1} PUYZ + \tfrac{1}{2} U^2 \right) + B^2 \Sigma YZE(U - E) = 0, \tag{15}$$

where

$U = u(x,t)/u_0, \quad P = p(x,t)/\rho_0 u_0^2,$

$\xi = (\delta/\lambda)\, x, \quad B = B_z/B_{RMS}, \quad \delta = B_{RMS}^2 \sigma_0 y_0 z_0 \lambda / m, \quad B_{RMS} = 2^{-\frac{1}{2}} B_m,$

$Y = y/y_0, \quad Z = z/z_0, \quad \Sigma = \sigma/\sigma_0 = \Sigma(U, P), \quad E = u_\lambda / u_0, \quad m = \rho_0 u_0 y_0 z_0.$

B_m is the maximum value of B in the flux wave, and zero subscript refers to inlet value.

These equations are now in a form in which they can be solved for velocity U and pressure P if we know the flux B, the conductivity σ and the cross-section y, z. For the travelling-wave converter we can specify B. The cross-section is generally assumed constant, although we shall look briefly at the effect of a variation in cross-section later.

The most difficult quantity is the conductivity which varies quite rapidly with temperature and to a lesser degree with pressure. Typical relations for equilibrium conditions are shown later in the table on optimization. There is experimental evidence though that equilibrium conditions do not apply in combustion flames and that the expansion process is too fast for the equilibrium to be established. There is also evidence that the electron temperature will depart from the gas temperature when a heavy electrical current flows. As a consequence, equilibrium relations are suspect in our context. The linearized approximation to conductivity used later on should, however, at least provide a general indication of the effect of conductivity variation with flow conditions.

In order to solve the equations only small perturbations are considered in the following sections.

4. Uniform duct

For a uniform duct the cross-section remains constant and we have

$$Y = Z = 1.$$

Hence equations (14), (15) become

$$\frac{\partial U}{\partial \xi}+\frac{\partial P}{\partial \xi}+\Sigma B^2(U-E) = 0, \tag{16}$$

and

$$\frac{\partial}{\partial \xi}\left(\frac{\gamma}{\gamma-1}PU+\tfrac{1}{2}U^2\right)+\Sigma B^2E(U-E) = 0. \tag{17}$$

If we are interested in the deviations U_1 and P_1 of U and P from their values for $x = 0$, i.e. from U_0 and P_0, these equations can be simplified. We have then

$$U = U_0+U_1(x,t), \quad P = P_0+P_1(x,t).$$

For the electrical conductivity we have

$$\Sigma = \Sigma_0+\Sigma_1,$$

where Σ_1 is to be derived from the general temperature and pressure dependent conductivity function $\sigma(T,p)$. Expanding the conductivity function about σ_0 we have

$$\sigma(T,p) = \sigma_0+\sigma_T T_1+\sigma_p p_1+\text{(higher-order terms)},$$

where σ_T and σ_p are the partial derivatives of σ with respect to T and p evaluated at (T_0,p_0). Introducing normalized quantities this gives

$$\Sigma_1 = \sigma_1/\sigma_0 = K_P P_1+K_U U_1+\text{(higher-order terms)},$$

where

$$K_P = \Sigma_T \gamma M_0^2 U_0+\Sigma_P, \quad K_U = \Sigma_T,$$

$$\Sigma_T = \sigma_T T_0/\sigma_0, \quad \Sigma_p = \sigma_p \rho_0 u_0^2/\sigma_0.$$

The two basic flow relations can now be expressed in terms of their deviations from their values at the duct entry and putting $\partial/\partial\xi = D$, we get

$$[D+B^2(1+C_U)]\,U_1+(D+C_P)\,P_1 = -B^2(U_0-E)+\text{(higher-order terms)}, \tag{18}$$

$$\left[\frac{\gamma}{\gamma-1}P_0D+B^2(1+C_U)\,E\right]U_1+\left[\frac{\gamma}{\gamma-1}U_0D+C_PE\right]P_1 = -B^2E(U_0-E) + \text{(higher-order terms)}, \tag{19}$$

where

$$C_U = K_U(U_0-E), \quad C_P = K_P(U_0-E).$$

For practical operating conditions two simplifications may be allowed:

(*a*) higher-order terms can be neglected if we consider only small changes in P_1 and U_1 and as long as the interaction terms on the right-hand side are small;

(*b*) we may consider only the average value of B^2 initially and may allow for sinusoidal variations later using superposition.

The relations for average values then become

$$(D+1+C_U)\,U_1+(D+C_P)\,P_1=-(1-E), \tag{20}$$

$$\left[\frac{\gamma}{\gamma-1}P_0D+D+(1+C_U)\,E\right]U_1+\left(\frac{\gamma}{\gamma-1}D+C_PE\right)P_1=-E(1-E), \tag{21}$$

where $$U_0=1,\quad B_0^2=B_m^2/2B_{RMS}^2=1.$$

Solving these relations we get

$$\left.\begin{aligned}U_1&=\lambda^{-1}U_0'(\epsilon^{\lambda\xi}-1)\approx U_0'\xi \quad (22)\\ P_1&=\lambda^{-1}P_0'(\epsilon^{\lambda\xi}-1)\approx P_0'\xi \quad (23)\end{aligned}\right\}\quad \text{for}\quad \lambda\xi\ll 1,$$

where

$$\lambda=\frac{\gamma-(\gamma-1)\,E}{(1/M_0^2)-1}+\frac{(\gamma-1)\,(1-E)}{(1/M_0^2)-1}\times\{-\Sigma_T\,[E+(1-E)\,M_0^2]-\Sigma_p[(1-E)+1/(\gamma-1)\,M_0^2]\},$$

$M_0^2=(\gamma P_0)^{-1}=u_0^2/a_0^2$, where a is the velocity of sound, and the derivatives of U_1 and P_1 at $\xi=0$ are

$$U_0'=\frac{[\gamma-(\gamma-1)\,E]\,(1-E)}{(1/M_0^2-1)},\quad P_0'=\frac{[\gamma P_0-(\gamma-1)\,E]\,(1-E)}{(1/M_0^2-1)}.$$

We have also $\Sigma_T>1$, $-1<\Sigma_P<0$ for partially ionized gases.

The results obtained are interesting and raise a number of issues. Whether we allow for electrical conductivity variations or not the initial rates of change in velocity and pressure remain unaltered. Hence the conductivity variations are only effective after some flow changes have taken place. The solutions for constant conductivity correspond closely with those for a d.c. conduction type plasma converter as discussed by Messerle & Morrisson (1962).

The effect of conductivity variation is to reduce the interaction of the flow with the magnetic field downstream. This is apparent when considering the reduction of the root due to the relatively large value of Σ_T for actual conductivity functions.

The solutions become singular for $M_0=1$ which indicates the possibility of producing choked conditions in a uniform duct. It shows also that small perturbations can have drastic effects once the flow approaches the velocity of sound.

Choking can be prevented by the change in electrical conductivity for a very large value of the conductivity parameter Σ_T. In that case the conversion or interaction dies away along the duct and the velocity settles down to an asymptotic value. This can be deduced directly from relation (22), since the characteristic root changes sign for large Σ_T and the velocity limit becomes

$$U_\infty=U_0+U_0'/\lambda\begin{cases}=U_{\max} & \text{for}\quad M_0<1,\\ =U_{\min} & \text{for}\quad M_0>1.\end{cases}$$

This effect is really undesirable from the practical angle, since it sets a limit to the energy that can be extracted from the gas. Luckily, it is found that Σ_T becomes less critical for higher temperatures. Conditions producing non-equilibrium ionization will also reduce the effective value of Σ_T.

Choking can also be avoided by changing the cross-section along the duct. In the subsonic case considered here the velocity tends to increase in the constant area duct and this is prevented by having a diverging duct. The shape can be designed to provide, for example, constant velocity or constant Mach number along the duct. The general form of the duct depends, however, on the loading and any electrical load change away from the design load requires a new duct shape, if a particular velocity distribution is desired.

As an example, consider a constant velocity for the load corresponding to any given E. If the width y of the duct is held constant and if only z is varied, an approximate relation for z can be deduced from the linearized flow equations. The required shape becomes in the case of constant σ

$$y = \text{const.}, \quad z = z_0 + z_1,$$
$$\frac{z_1}{z_0} = \left(\frac{\gamma}{\gamma-1} - E\right)\frac{1}{E}(1-\epsilon^{\lambda_z \xi}) \approx [\gamma - (\gamma-1)E](1-E)\xi M_0^2, \tag{24}$$
$$\lambda_z = -E(1-E)(\gamma-1)M_0^2.$$

Hence the shape depends on E and the entrance Mach number. When conductivity variations are allowed for, the shape becomes a function of inlet temperature as well.

Flow and choking conditions are also modified by the heat loss from the plasma. For small-sized converters this loss can be quite severe; however, it is expected to be reasonable for converters in the megawatt region.

5. Power flow relations

The electrical output at any position along the duct was given by relation (5). Introducing the small perturbation relations, it is possible to express the electrical output also in terms of a steady reference quantity W_{el_0} and a perturbation component W_{el_1}. To show this, substitute (u_0+u_1) for u into (5), i.e.

$$dW_{el} = B_z^2\sigma yz[u_\lambda(u_0-u_\lambda)+u_\lambda u_1]\,dx = dW_{el_0}+dW_{el_1}, \tag{25}$$

where $$dW_{el_0} = B_z^2\sigma yzu_\lambda(u_0-u_\lambda)\,dx = B_z^2\sigma yzu_0^2E(1-E)\,dx$$

is the incompressible flow power, and

$$dW_{el_1} = B_z^2\sigma yzu_\lambda u_1 dx = B_z^2\sigma yzu_0^2 EU_1 dx$$

is the perturbation effect due to compressibility. The component of power due to incompressible flow is for constant σ, so that

$$W_{el_0} = \int_0^\lambda dW_{el_0} = \tfrac{1}{2}B_m^2\sigma\lambda\, yzu_0^2E(1-E).$$

Introducing the inlet Mach number M_0, this becomes

$$W_{el_0} = W_a M_0^2 E(1-E) = -s(1-s)^{-2}W_a M_0^2, \tag{26}$$

where $W_a = \frac{1}{2}B_m^2\sigma\lambda yza_0^2$, $E = (1-s)^{-1}$, and $s = u_\lambda^{-1}(u_\lambda - u_0)$ is the slip factor.

For the perturbation component we have, if σ is constant,

$$W_{el_1} = \int_0^\lambda dW_{el_1} = \sigma yzu_0^2 E\int_0^\lambda B^2U_1 dx.$$

This contains a number of second and fourth harmonic terms, but they all average out and we get finally over one wavelength of duct

$$W_{el_1} = \tfrac{1}{4}B_m^2\sigma yzu_0^2 EU_0'\delta\lambda = \tfrac{1}{2}W_a M_0^2 EU_0'\delta. \tag{27}$$

The total power is then

$$W_{el} = W_a M_0^2 E[(1-E)+\tfrac{1}{2}U_0'\delta]. \tag{28}$$

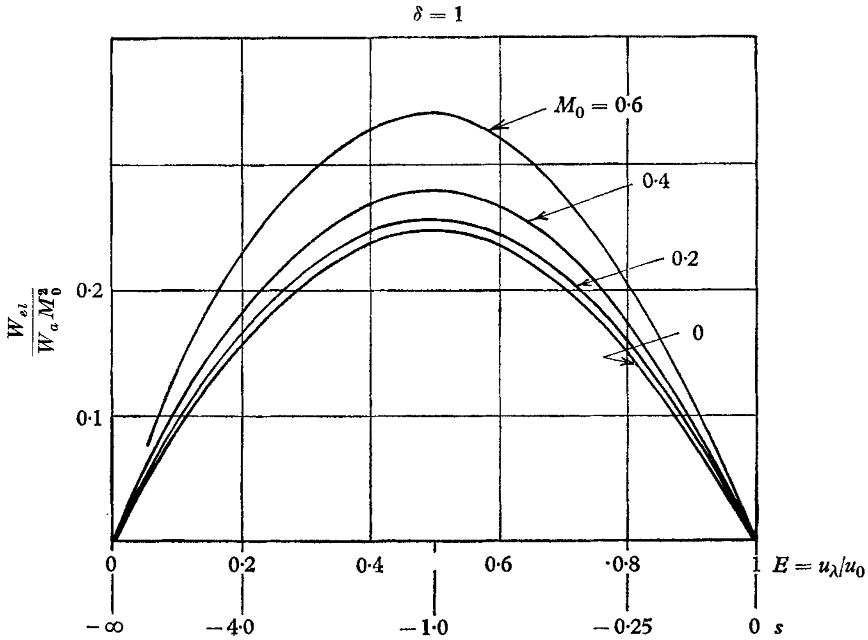

FIGURE 3. Electrical power output plotted against E for constant δ (= 1) and showing the effect of increased inlet Mach number M_0.

This is the electrical power allowing for compressibility. The perturbation effect depends on the change of velocity and the parameter δ, which is the only quantity related to the density of the plasma.

Relation (28) can be put into different forms and introducing slip s, it becomes

$$W_{el} = W_a . M_0^2 \frac{(-s)}{(1-s)^2}\left[1+\frac{\{\gamma-(1-s)^{-1}(\gamma-1)\}}{2(1-M_0^2)}M_0^2\delta\right]. \tag{29}$$

For small Mach number M_0 and for small δ we find that the compressibility has little effect on the overall power conversion process, and it is of interest to have a closer look at the effect of some of the critical parameters. Typical results are plotted in figures 3–5. The effect of increased inlet Mach number is shown in figure 3. The perturbation is much more pronounced as we approach sound velocity and choking must be watched. For a relatively large perturbation power the approximations used here obviously fail and the curves only provide a general guide. Digital computer checks, however, have shown that the linearized relations are very good for practical operating regions as long as they are restricted to subsonic flow. For supersonic flow greater care has to be taken as shown by Messerle & Morrison (1962).

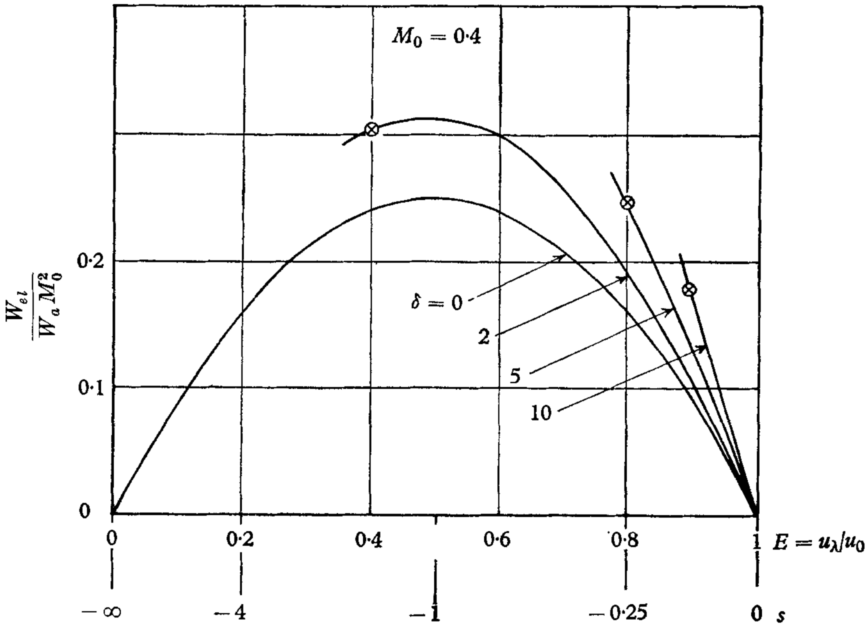

FIGURE 4. Electrical power output plotted against E for constant M_0 (= 0·4) and showing the effect of changing δ. ⊙ = choke points.

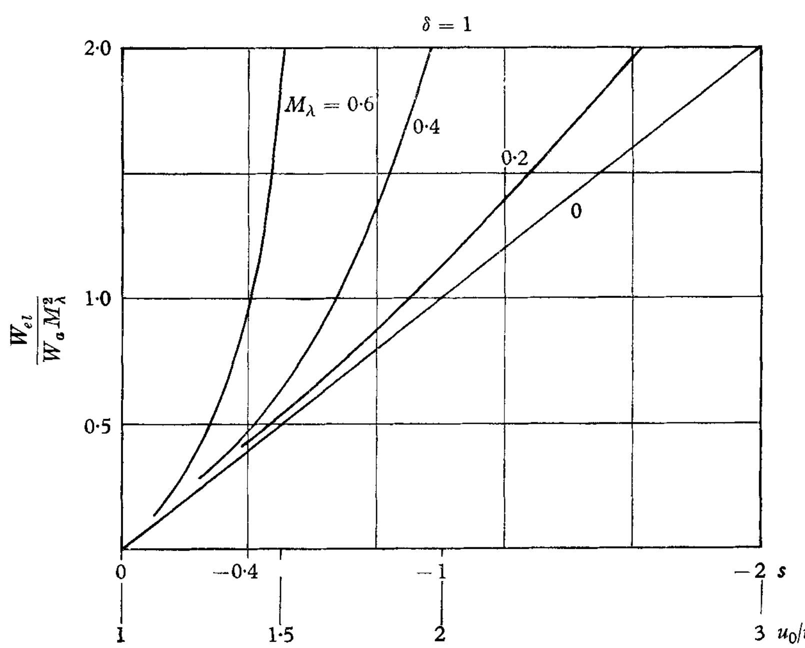

FIGURE 5. Electrical power plotted as function of s for constant δ (= 1) and showing the effect of raising the wave velocity.

The effect of changes in the length parameter δ are shown in figure 4. An increase in δ here corresponds primarily to an increase in the electrical conductivity, or a decrease in the plasma density.

When holding the wave velocity constant, the power increases indefinitely with negative values of s as shown in figure 5. This conclusion will have to be modified later when allowing for variability of σ.

6. Time-dependent flow variations

So far, the effect of the sinusoidal flux components has been ignored. For a complete analysis the general one-dimensional flow equations must be used. Relations (6) to (8) apply with

$$\frac{d}{dt} = \frac{\partial}{\partial t} + u\frac{\partial}{\partial x}.$$

If we consider again only small variations, these relations, after elimination of all variables except p_1 and u_1, reduce to

$$(D_t + u_0 D_x + \rho_0^{-1}B^2\sigma_0)\,u_1 + \rho_0^{-1}D_x p_1 = -\rho^{-1}B^2\sigma(u_0 - u_\lambda), \tag{30}$$

and

$$[(c_p T_0 + u_0^2)\,D_x + u_0 D_t + \rho_0^{-1}B^2\sigma_0 u_\lambda]\,u_1 + \rho_0^{-1}\left[\frac{1}{\gamma-1}D_t + \frac{\gamma}{\gamma-1}u_0 D_x\right]p_1 = -\rho^{-1}B^2\sigma u_\lambda(u_0 - u_\lambda), \tag{31}$$

where $D_x = \partial/\partial x$ and $D_t = \partial/\partial t$. At this stage the electrical conductivity is assumed to be constant and only a very short duct length, or small interaction, is considered.

The time variations are sinusoidal since both forcing functions on the right-hand side of equations (30), (31) depend on B_z^2, where

$$B_z^2 = \tfrac{1}{2}B_m^2[1 + \cos 2(\omega t - 2\pi\lambda^{-1}x)].$$

The constant term has been discussed earlier, hence only the double-frequency term is of relevance. The differential operators so become, for the double-frequency variations,

$$D_x = -4\pi i\lambda^{-1} \quad \text{and} \quad D_t = 2\omega i. \tag{32}$$

If we ignore higher harmonics in the terms containing B^2, we have to replace B^2 by $B_m^2/2$ on the left-hand side of equations (30) and (31). As follows the characteristic determinant for equations (30) and (31) has the value

$$\Delta = \frac{1}{\rho_0(\gamma-1)}\left\{D_t^2 + \frac{B_m^2\sigma}{2\rho_0}D_t + \left[2u_0 D_t + (\gamma u_0 - (\gamma-1)\,u_\lambda)\frac{B_m^2\sigma}{2\rho_0}\right]D_x + u_0^2(1 - 1/M_0^2)\,D_x^2\right\}, \tag{33}$$

and this leads to

$$\Delta = \left(\frac{4\pi a_0}{\lambda}\right)^2\frac{1}{\rho_0(\gamma-1)}\left[M_0^2 - M_0\left(2M_\lambda - i\frac{\gamma\delta_a}{4\pi}\right) + \left(M_\lambda^2 - iM_\lambda\frac{\gamma\delta_a}{4\pi} - 1\right)\right], \tag{34}$$

where $\delta_a = B_m^2\sigma\lambda/2\rho_0 a_0$ = interaction parameter,

and $M_\lambda = u_\lambda/a_0$.

The characteristic determinant now allows us to study the fluid flow response to sinusoidal perturbation. Its roots determine resonance conditions if they exist.

The two characteristic roots are

$$M_0 = M_\lambda - i\frac{\gamma\delta_a}{8\pi} \pm \left(1 - \frac{\gamma^2\delta_a^2}{64\pi^2}\right)^{\frac{1}{2}}, \tag{35}$$

which indicate a resonance effect for small values of δ_a. For very small interactions, or for zero field, singular solutions arise when

$$M_0 = M_\lambda \pm 1,$$

or $$u_{0r} = u_\lambda \pm a_0 = \text{resonance velocity.}$$

Hence for small interaction the sinusoidal flow disturbance can be critically amplified if the wave velocity lags, or leads the plasma velocity by the velocity of sound.

It is important to realize that the relative sound velocity u_{0r} becomes critical for sinusoidal interaction and not the absolute sound velocity a_0 which is critical for the steady interaction component of the travelling wave. For design purposes, therefore, both conditions have to be considered since the travelling flux wave produces both types of interaction.

For practical conditions the plasma velocity can be expected to be below that of sound. In that case both critical wave velocities can be expected to be outside the operating region, since one would be above that of sound and the other one negative. The resonance conditions for small interaction, therefore, would lie outside the region of the operating velocities. For larger interaction the characteristic roots indicate that the resonance is damped and the resonance conditions change. We have then

$$|u_\lambda - u_{0r}| < a_0,$$

or the resonance velocity approaches the wave velocity for large interaction. In practice it is found that the interaction parameter δ_a is of the order of 5 or less. Thus actual interaction is usually low and the resonance effect should be noticeable.

7. Discussion

General performance characteristics were shown in figures 3–5, and optimum operating conditions can be deduced quite readily for incompressible flow. When considering generation of electricity, there is usually a given practical maximum plasma velocity. For this velocity it is then desired to find the travelling-wave velocity which leads to maximum power output.

In terms of plasma velocity the electrical output is (ignoring the perturbation term)

$$W_{el_0} = -s(1-s)^{-2} W_a M_0^2. \tag{36}$$

This is a maximum when

$$s = -1, \quad \text{or} \quad u_\lambda = \tfrac{1}{2}u_0,$$

and then $$W_{el\max} = \tfrac{1}{4} W_a M_0^2.$$

Hence, maximum power for fixed plasma velocity is obtained when the travelling-wave velocity is half that of the plasma. This is generally called the optimum condition; however, for a fixed wavelength λ the wave speed can be changed only by changing the frequency f.

If the frequency is kept constant, the wavelength must change as u_λ changes and the output becomes

$$W_{el} = -s(1-s)^{-3}W_f, \tag{37}$$

since
$$\lambda = u_\lambda/f = (1-s)^{-1}u_0/f,$$

and
$$W_f = \tfrac{1}{2}u_0^3 B_m^2 yz\sigma/f = W_a M_0^2 u_0/u_\lambda.$$

The maximum occurs now for $s = -\frac{1}{2}$, i.e. when the wave velocity is 2/3 of the plasma velocity and the maximum output is

$$W^f_{el\max} = \tfrac{4}{27}W_f.$$

This maximum is less than the one obtained for constant λ, if we set as reference a $\lambda_r = u_0/f$. In that case

$$W_{el\max} = \tfrac{1}{4}W_{ar}M_0^2 = \tfrac{1}{4}W_f > W^f_{el\max}.$$

In practice both f and λ are fixed. The frequency is determined by the system and then λ can be determined if the optimum plasma velocity is known. The optimum plasma velocity is determined by the velocity dependence of the electrical conductivity which overrides the rise in power density with velocity, due to the velocity terms in relations (36) and (37).

The electrical conductivity in a converter duct depends on the local conditions at any point. As the plasma velocity is raised, the local temperature and pressure drop, hence conductivity, changes too. It drops in practice because of its sensitivity to temperature variations and the resulting effect on power density can be studied by using either theoretical or experimental expressions for it.

The theoretical relationship between conductivity and the thermodynamic quantities is usually based on Saha's equation which applies for equilibrium conditions. Whether equilibrium conditions exist in a converter duct is questionable as discussed earlier. Consequently, any deductions based on equilibrium conditions only provide a rough guide.

Using Saha's equation, Ralph (1961) has shown for the direct current plasma converter that optimum Mach numbers are restricted to about 0·5 for gases seeded with alkali metals. Using a more flexible approximation, Swift-Hook & Wright (1962) have shown that the Mach number may be somewhat higher depending on the ratio of specific heats. In the case of the travelling-wave converter these conclusions can be applied directly for the constant wavelength conditions. For fixed wave frequency, however, the optimum Mach number can be up to 50% higher. The optimum conditions are compared in table 1 for conversion conditions per unit wavelength. In the constant frequency case a higher Mach number becomes permissible because of the lengthening of the duct as wave velocity goes up. In practice this leads to the question of multiple wavelengths and a comparison of multiple wavelength converters at low velocity with high velocity converters using a smaller number of wavelengths.

Multiple wavelengths are actually desirable for practical operating reasons. The sudden transition from a field free region into the field region of a converter produces a disturbed region in the plasma with considerable eddy current losses unless the transition is graduated. Blake (1957) has shown for liquid-metal travelling-wave pumps how the transition can be smoothed out when using multiple wavelengths. The method is to allow the field amplitude to build up and reach a maximum over the middle section of the converter covering several wavelengths.

For a plasma converter operating at Mach numbers of the order of 0·5 the wavelengths for ordinary power frequencies are of the order of 5 m. Such lengths will restrict the application of multiple wavelength principles; however, a superposition of several wavelengths of slightly differing individual length might provide a solution as suggested by Sudan (1963).

	$\sigma = b\,\dfrac{T^{\frac{3}{4}}}{p^{\frac{1}{2}}}\,\epsilon^{c/T}$, b and c = const.	$\sigma = \sigma_0 \left(\dfrac{T}{T_0}\right)^b \left(\dfrac{p}{p_0}\right)^c$
Constant λ	$M_{opt} = \dfrac{2G}{\gamma-1}$ $G = (g+2j)+[(g+2j+1)^2+8gj]^{\frac{1}{2}}$, $g = \dfrac{2c}{T_0}$, $j = \dfrac{3\gamma}{2(\gamma-1)}-\dfrac{9}{4}$	$M_{opt} = [(b+1)(\gamma-1)-(c+1)\gamma]^{-\frac{1}{2}}$
Constant f	$M_{opt} = \dfrac{2H}{\gamma-1}$, $H = (g+2q+1)+[(g+2q+2)^2+8gq]^{\frac{1}{2}}$ $q = \dfrac{3\gamma}{2(\gamma-1)}-\dfrac{11}{4}$	$M_{opt} = 2^{\frac{1}{2}}[(b+1)(\gamma-1)-(c+1)\gamma]^{-\frac{1}{2}}$

TABLE 1. Optimum Mach numbers.

In conclusion it must be mentioned that the problem of the energy required to produce the travelling wave still remains to be solved. Using present-day techniques, the inductance of the field structures becomes very large. This leads to excessive reactive power requirements. However, the problem may be overcome by working at higher frequencies or by raising the operating temperatures into regions beyond 3000 °K as considered in most applications to date.

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